1 Optimization problem

Balancing a disaggregated table, respecting constraints of coherence to original table and including exogenous information.

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\begin{aligned}
&\textbf{Sets} \\
&\qquad S = \text{New sectors} \\
&\qquad R = \text{Regions} \\
&\qquad F = \text{Factors of production} \\
&\qquad C = \text{Consumption category} \\
&\text{Sub-sets of } \textbf{sectors} \\
&\qquad S_N = \text{New sectors} \\
&\qquad S_P = \text{Residual sectors} \\
&\qquad S'_P = \text{Parent sectors} \\
&\qquad S_S = \text{Stable sectors} \\
&\qquad S_{PN} = \text{Parent and new sectors}
\end{aligned} \tag{1}
$$

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\begin{aligned}
\min_{Z,Y,V} \quad & \sum_{rf,rt} \Big(\sum_{sf,st \in S_N} H(Z, Z^0) + \sum_{sf \in S_S, st \in S_P} H(Z, Z^0) \Big) + \sum_{f, st \in S_N, rt} H(V, V^0) \\
\text{s. t.} \quad & \sum_{sf \in S_{PN}} Z\alpha(sf, sf') = Zold && \forall sf' \in S'_p, \forall st \in S_s, \forall rf \in R, \forall rt \in R \\
& \sum_{sf \in S_{PN}} \sum_{st \in S_{PN}} Z\alpha(sf, sf')\alpha(st, st') = Zold && \forall sf' \in S'_P, \forall st' \in S'_P, \forall rf \in R, \forall rt \in R \\
& \sum_{st \in S_{PN}} Z\alpha(st, st') = Zold && \forall sf \in S_S, \forall st' \in S'_P, \forall rf \in R, \forall rt \in R \\
& \sum_{sf \in S_{PN}} \sum_{st \in S_{PN}} Z\alpha(sf, sf')\alpha(st, st') = Zold && \forall sf' \in S'_P, \forall st' \in S'_P, \forall rf \in R, \forall rt \in R \\
& \sum_{sf \in S_{PN}} Y\alpha(sf, sf') = Yold && \forall sf' \in S'_P, \forall c \in C, \forall rf \in R, \forall rt \in R \\
& \sum_{st \in S_{PN}} V\alpha(st, st') = Vold && \forall st' \in S'_P, \forall f \in F, \forall rt \in R \\
& \sum_{rt \in R} \Big(\sum_{st \in S} Z + \sum_{c \in C} Y \Big) = X(sf, rf) && \forall sf \in S_{PN}, \forall rf \in R \\
& \sum_{rf \in R} \Big(\sum_{sf \in S} Z + \sum_{f \in F} V \Big) = X(st, rt) && \forall st \in S_N, \forall rt \in R \\
& \sum_{sf \in S} X\alpha(sf.sf') = Xold(sf', rf) && \forall sf' \in S_P, \forall rf \in R \\
& \sum_{rt \in R} \Big[\Big(\sum_{st \in S} Z + \sum_{c \in C} Y \Big) \beta(rf, rt) \Big] = T && \forall sf \in S_N, \forall rf \in R
\end{aligned} \tag{2}
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Definition of operators

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\begin{aligned}
& H(x, y) = (x + \epsilon) ln[(x + \epsilon)/(y + \epsilon)] && \text{where } \epsilon = 1e^{-6} \\
& \alpha(sf, sf') = 1 \text{ if } \textit{sf'} \text{ is parent of } \textit{sf}\text{, else } 0 \\
& \beta(rf, rt) = 1 \text{ if } rf \neq rt\text{, else } 0
\end{aligned} \tag{3}
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